High Dynamic Control of Elastic Drive Systems

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Vibrations – an important design aspect

Clonk

Judder

Shuffle

Whoop

Boom

Chatter

Rattle

1) http://www.openclipart.org
2) http://www.howstuffworks.com
3) http://www.zf.com
Outline

- Elastic Drive Systems
- Model Predictive Control (MPC)
- Application example
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Introduction to Elastic Drive Systems I

- Encountered in various fields:
  - Papermaking
  - Wind turbines
  - Vehicle traction (powertrains)
  - Test beds, …

1) Papermill at http://www.alibaba.com
3) http://www.windpowersaving.com
Introduction to Elastic Drive Systems II

- Characterized by concentrated inertias connected by elastic shafts
- Oscillatory behavior
- Equations of motion can be derived from angular momentum balance

Equations of Motion (Simple Example)

Angular momentum balance:
\[ I_E \ddot{\varphi}_E = T_E - T_S \]

Linear shaft equation:
\[ T_S = k \Delta \varphi + d \Delta \dot{\varphi} \]

State space model:
\[
\begin{bmatrix}
\dddot{\varphi}_E \\
\ddot{\varphi}_D \\
\Delta \ddot{\varphi}
\end{bmatrix} =
\begin{bmatrix}
\frac{1}{I_E} \begin{bmatrix}
-k & -d & d \\
-k & d & -d \\
1 & -1 & 0
\end{bmatrix} & 0 & 0 \\
0 & \frac{1}{I_D} & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\varphi_E \\
\varphi_D \\
\Delta \varphi
\end{bmatrix}
+ \begin{bmatrix}
\frac{1}{I_E} \\
0 \\
0
\end{bmatrix} T_D + \begin{bmatrix}
\frac{1}{I_E} \\
0 \\
0
\end{bmatrix} T_E
\]

\[
y = T_S = \begin{bmatrix} d & -d & k \end{bmatrix} \begin{bmatrix}
\varphi_E \\
\varphi_D \\
\Delta \varphi
\end{bmatrix}^T
\]

\[
\dot{x} = Ax + Bu + Ez \\
y = Cx
\]
Vibration Reduction vs. Vibration Induction

Goal for most elastic drive systems (e.g. industrial drives):

- Reduce unwanted oscillations that have negative effect on performance

Goals for test beds:

- Reduce unwanted oscillations not occurring under normal operating conditions
- Induce oscillations occurring under normal operating conditions
Realistic testing requires that oscillations and their sources are well understood, i.e. by dynamic models of:

- Test bed vibrations
- Vibrations during vehicle operation

Dynamic models can then be used to

- Predict future onset of oscillations
- Parametrise a predictive controller that either suppresses or induces vibrations
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Model Predictive Control (MPC) uses:
• an internal dynamic model of the plant
• a history of past control moves and
• an optimization cost function

At each time step:
• Plant state is sampled
• Future output is predicted
• Cost minimizing control strategy is computed
• first step of the control strategy is implemented (rest discarded)
Everyday MPC – Driving a Car
Advantages of model predictive control:
• Effectively deal with large time delays and high-order dynamics
• Possibility to directly incorporate constraints
• Drive plant close to its limits

Drawbacks of model predictive control:
• Higher complexity than classical controllers
• Computational effort (especially when considering constraints)
Fundamental Equations of MPC I

- Cost function to be minimized

\[ J = \sum_{i=1}^{N_p} \sum_{j=1}^{n_y} Q_{i,j} (r_j(k+i|k) - y_j(k+i|k))^2 + \sum_{i=0}^{N_c-1} \sum_{l=1}^{n_u} R_{i,l} \Delta u_i^2(k+i|k) \rightarrow \text{Min.} \]

- Find optimal control sequence (=minimum of cost function) by setting first derivative of \( J \) with respect to \( \Delta u \) to zero

\[ \frac{\partial J}{\partial \Delta u} = 0 \]
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Engine Test Bed Overview

Dynamometer reference torque (control input)

Measured shaft torque

Torque controller

Frequency converter (FC)

Electric air gap torque

Internal combustion engine (ICE)

Load machine (dynamometer)

Torque sensor

Connecting shaft (CS)

Measured shaft torque

$T_D$

$T_S$

$T_{el}$
Rotor Model

- Mechanical rotor model identified by exciting rotor using a chirp signal
- Modeled by three mass oscillator (two nonzero resonance frequencies)
- Good coincidence of measurement and simulation
- Superharmonic resonances due to imperfect excitation (truncation)
Reference Tracking (Measurement)

- Operation at first rotor resonance frequency possible
- Disturbance compensation highly beneficial
Predictive Disturbance Compensation

- Combustion torque acts as periodic disturbance
- Combustion torque can not be directly measured on a production engine
- Estimation e.g. from measured engine acceleration and measured shaft torque via angular momentum balance

\[ I_E \dot{\omega}_{EB} = T_{com} - T_S \]
Sine tracking with disturbance compensation

- Torque in Nm
- $T_D$ in Nm
- $n_{EB}$ in rpm

Controller engaged
Dist. comp. engaged

Time in s

1.45 1.5 1.55 1.6 1.65 1.7 1.75 1.8
The problem of constrained MPC

Graph showing the time evolution of $T_{d}$ and $T_{e}$ in Nm.
Goal: Get close to limit but do not exceed it!

In practice all processes are subject to constraints

• Actuators
• Safety Limits
• Operating conditions

Advantages of model predictive control:

• Constraints can be directly incorporated

1) http://www.openclipart.org
Find minimum of cost function $J(x)$ with respect to $x$

$$J = \frac{1}{2} x^T E x + x^T F$$

subject to inequality constraints

$$M x \leq \gamma$$

leads to a quadratic programming problem. There are many reliable QP algorithms, for real-time implementation computation time is critical!
Magnetic Levitation:

- Electromagnet levitates steel ball
- Track ball position to a desired trajectory
Constraint Handling in Action

Unknown constraint encountered

Constraint handling enabled
• Solving full quadratic programming problem is often infeasible in realtime!

• Explicit MPC (eMPC)
  • Offline pre-computation of ALL(!) constrained optimal MPC control laws
  • Unsuitable for long prediction horizons, number of solutions grows exponentially according to $3^{N_C-1}$
    • For $N_C=2$ there are 8 constrained solutions
    • For $N_C=10$ there are already 59048 constrained solutions!
A new approach for fast MPC (Ts~0.1ms)

- Considering constraints over a limited constraint horizon
  - Find most severely violated constraint and add to set of active constraint
  - Update solution based on active set
  - Repeat until end of constraint horizon or until no constraints are violated any more
- Approximates the QP problem (not necessarily optimal)
- Works well for elastic drive systems
• Active constraint handling can dramatically improve performance!

• Constraints actively considered in real time at 10kHz sampling frequency
Conclusion

• Model predictive control enables high dynamic control of elastic drive systems

• Controller requires high fidelity plant model

• Combustion torque disturbance compensation is highly beneficial

• Constraints can be actively incorporated despite high sampling frequency (especially beneficial in weakly damped scenarios)

• Easily transferable to various types of elastic drive systems
Outlook: Taking HiL to the Crank Angle Level

Diagram showing the integration of hardware under test (ICE) and real-time control unit within a simulated test environment.
Thank you for your attention!