The present paper gives a review on the research on smart structures based on piezoceramics, which has been performed at the Institute of Technical Mechanics of the Johannes Kepler University Linz, Austria, during the last twenty years.

Particularly, own contributions in the following fields are reviewed: Accurate electromechanically coupled structural modeling; dynamic shape control by piezoelectric actuation and sensing; sensor networks for measuring structural behaviour and damage.
A novel fundamental result of eigenstrain-theory (1)


**Displacement tracking problem:**

Assume that the initial past history, the geometrical and mech. parameters, the body forces and the boundary data are known. **Seek a transient field of tensorial actuator-stresses, such that the displacement** \( u \) **satisfies the following relation**

\[
    u = z
\]

**everywhere in the viscoelastic body and for all times, where** \( z \) **is a given, sufficiently smooth displacement field (“displacement field, which shall be tracked”).**
A novel fundamental result of eigenstrain-theory (2)

Consistency:

a) Initial past histories of $u$ and $z$ are equal.

b) The kinematic b.cs. of $u$ and $z$ are equal.

Solution: An eigenstrain-actuation represents a solution of the dynamic displacement tracking problem, if the actuator tensor satisfies the following equilibrium condition at all times and everywhere:

$$ \text{Div} \ S_A + b^* = 0 $$

with

$$ b^* = b - \rho \dot{z} + \text{Div} \left( G_0 \ [\text{sym} \ \nabla z] + \int_0^\infty \dot{G}(s) \ [\text{sym} \ \nabla z \ (t-s)] \ ds \right) $$

$$ \hat{s}^* = \hat{s} - (G_0 \ [\text{sym} \ \nabla z] + \int_0^\infty \dot{G}(s) \ [\text{sym} \ \nabla z \ (t-s)] \ ds) \ n $$
Numerical example for Shape Control:

- **Zero-Tracking:** $z = 0$
- Plane strain
- Computation of quasi-static stress by ABAQUS (525 plane strain elements of type CPE4R)
- **Thermal Actuation** (anisotropic heat conduction)

Numerical example for Shape Control (2)

Frequency response

Jump response

Collocated Actuators and Sensors ("Natural output"): M. Krommer, H. Irschik: *Sensor and actuator design for displacement control of continuous systems*, Smart Structures and Systems, 2007

Piezoelectricity: “...The generation of electric charge in a substance by a mechanical stress that changes its shape, and a proportional change in the shape of a substance when voltage is applied.” Encyclopædia Britannica

Sensors: “...the generation of electric charge in a substance by a mechanical stress”

Actuators: “...change in the shape of a substance when voltage is applied.”
“The young brothers Jacques and Pierre Curie announced their discovery of the piezoelectric effect to the French Academy of Science on 2 August 1880. Yet, the use of the phenomenon outside the laboratory began only 35 years after its discovery during the first world war. Technological applications were not a main concern in the early study of piezoelectricity.” S. Katzir

A modern application: Piezo-Injectors

"Piezo-Injektoren: Neue Technik für saubere und sparsame Diesel- und Benzinmotoren"
Piezoelectric Elements for integration in vibrating mechanical structures:

piezoelektrische Keramik (Patch)

piezoelektrische Folien (PVDF)

piezoelektrische Stapelaktoren

Piezelektrische Fasermodule

Piezoelectric fiber (30 µm)

Quelle für Keramik: Invent (DLR); für Fasermodule: Fraunhofer Institut für Silikatforschung, Würzburg
Compensation of cantilever vibrations (1):

Galileos cantilever: First systematic study of simple structural elements

Additional actuators produce eigenstrains:
- Thermal,
- Piezoelectric,
- Magnetostrictive,
- Active tendons ...

Shape Control seeks for actuator distributions (shaped actuators), which can compensate strains and vibrations due to given forces or rigid body motions, if possible, completely.

Galileo Galilei: Discorsi e demostrazioni matematiche, intorno a due nuovo scienze, Leyden 1638

Abb.: I. Szabo, Geschichte der mechanischen Prinzipien, Bikhäuser 1987
Compensation of cantilever vibrations (2):

Compute quasi-static parts of mechanical stresses (bending moments!) due to inertial forces. This yields theoretical piezoelement distribution as a quadratic function in space.
Compensation of cantilever vibrations (3):

Realization of piezo-actuation: PZT (Blei-Zirkonat-Titanat) on aluminium-substrate

Continuous distribution:

Piezoelectric Patches:
Compensation of cantilever vibrations (4):

Distribution of piezoelectric patches:

Distribution of patches such that auxiliary loading of conjugate beam forms a region-wise self-equilibrating system (Mohr’s Analogy).

Compensation of cantilever vibrations (4):

3D-Finite-Element computation as a first validation:

Piezo-Actuation: Distribution according beam-theory

FE-computation: coupled, three-dimensional (3D) solid elements

Compensation of cantilever vibrations (5):

Realization of concepts of automatic control:

**Feedforward (FX-LMS)**
Anregung ist bekannt

- Feedforward: Measurement of disturbance; identification of plant;
- Feedback: disturbance-control with loop-shaping

**Feedbackward-Reglerentwurf**
Anregung ist nicht bekannt

Uncertainties:
- mech. system-parameters
- electr. system-parameters
- time-evolution of excitation

Compensation of cantilever vibrations (6):

Experimental validation (Laboratory):

- Support-excitation by Piezo-Stack

- Vibration compensation by Piezo-Patches:

http://www.lcm.at
Compensation of cantilever vibrations (7):
Compensation of cantilever vibrations (8):

Excitation by band-limited noise;
compensation in a frequency band including first and second eigenfrequency

Harmonic, resonant excitation in first eigenfrequency; time-domain

Problem:
Vibrations of the funnel: Heavy noise-load for patients and personal (above 80 dB); passive damping not sufficient.
Vibration-compensation in thin shells with a complex shape (2)

Application of strategy developed for beams and plates to the complex funnel:

Erstellung eines Finite-Elemente Modells

Modellabgleich mittels Laser Scanning Vibrometrie
Vibration-compensation in thin shells with a complex shape (3)

Computation of quasi-static stresses (here: 1. invariant of stress-tensor) due to inertial forces by FE  
—> Theoretical actuator-distribution  
—> Discretization by piezo-patches

Goal: Automatic reduction of the noise produced by ventilation devices

Example: Aktive Active noise-channellation in a heat ventilation air conditioning device of a car:

Piezo-Actuators for generating anti-noise by exciting vibrations in the light-weighted solid shell of the duct
Work in the last years (exemplary): Actuator and Sensor-Networks for Displacement-Tracking

Example: Frame (upper floor horizontally movable) with patch-network; tracking of second Eigenmode of clamped-clamped beam

A general concept for strain-type sensors (1)

\[ V : \nabla \cdot \sigma^{(aux)} + b^{(aux)} = 0, \]
\[ \partial B_{u_1} + \partial B_{\sigma} : \sigma^{(aux)} \cdot n = t^{(aux)} \]

Principle of virtual work for auxiliary quasi-static problem

\[ -\int_V \sigma^{(aux)} : \delta \varepsilon dV + \int_V b^{(aux)} : \delta u dV + \int_{\partial B_{u_1} + \partial B_{\sigma}} t^{(aux)} : \delta u dB = 0. \]
A general concept for strain-type sensors (2)

Sensor signal of a continuously distributed (continuous) strain-type sensor

\[ y(t) = \int_V \mathbf{S}_s(r) \cdot \varepsilon(r, t) dV \]

- \( \varepsilon = \text{sym}(\nabla \mathbf{u}) \) ... linearized strain tensor
- \( \mathbf{u} \) ... displacement vector
- \( \nabla \) ... invariant differential operator
- \( \mathbf{S}_s \) ... shape tensor defines the continuous distribution of the strain-type sensor
A general concept for strain-type sensors (3)

- Displacement of original problem is kinematically admissible \( \delta \mathbf{u} = \mathbf{u}(\mathbf{r}, t) \)

- Statically admissible stress tensor as sensor shape tensor \( \mathbf{S}_s(\mathbf{r}) = \sigma^{(aux)}(\mathbf{r}) \)

\[
\begin{align*}
V : & \quad \nabla \cdot \mathbf{S}_s(\mathbf{r}) + \mathbf{b}^{(aux)} = \mathbf{0}, \\
\partial B_{u_1} + \partial B_\sigma : & \quad \mathbf{S}_s(\mathbf{r}) \cdot \mathbf{n} = \mathbf{t}^{(aux)} \end{align*}
\]

Sensor measures work of the external auxiliary forces on the original deformation

\[
\begin{align*}
\gamma(t) &= -\delta \mathcal{W}^{(i, aux)}(\delta \mathbf{e} = \mathbf{e}, \sigma^{(aux)} = \mathbf{S}_s) = \int_V \mathbf{S}_s \cdot \delta \mathbf{e} dV \quad \leftrightarrow \quad \text{structural level} \\
&= \delta \mathcal{W}^{(e, aux)}(\delta \mathbf{u} = \mathbf{u}) = \int_V \mathbf{b}^{(aux)} \cdot \mathbf{u} dV + \int_{\partial B_\sigma} \mathbf{t}^{(aux)} \cdot \mathbf{u} dB \quad \leftrightarrow \quad \text{design level}
\end{align*}
\]
Some experimental results

Displacement sensor: Experiment - laser vs. piezoelectric

Nilpotent sensor: Analytical vs. Experiment
Sensor network design (1)

- Body divided into $n$ sub-domains: $V_i$, $i = 1, \ldots, n$, $\bigcup_{i=1}^{n} V_i = V$
- One sensor with constant intensity within each sub-domain: $\tilde{V}_i$, $\tilde{V}_i \subset V_i$
- A weight $S_{Si}$ is assigned to each sensor

**Signal of the sensor network**

$$\tilde{y}(t) = \sum_{i=1}^{n} S_{Si} \cdot \int_{\tilde{V}_i} \varepsilon(r, t) dV$$

→ Design of the network to minimize error signal

$$e(t) = \int_{V} S_{S} \cdot \varepsilon dV - \sum_{i=1}^{n} S_{Si} \cdot \int_{\tilde{V}_i} \varepsilon(r, t) dV$$

→ Optimization strategy strongly depends on structural problem!
Sensor network design (2)

- One storey frame structure
- Measurement of floor displacement with single-purpose sensor network
- Eight piezoelectric patches at each sidewall as a sensor network
Sensor network design (3)

→ Sensor design for one-storey frame structure

Floor displacement sensor

→ Proper superposition of nilpotent sensors

Nilpotent sensor

→ Sensor design for multi-storey frame structure by using these sensors for each storey
Sensor network design (4)

- Redundant three storey frame structure
- Measurement of 3rd floor displacement relative to harmonic ground excitation
- Three piezoelectric patches at each sidewall as a sensor network
- Analytical and 3D Finite Elements (Ansys)

→ Sensor network works in both cases due to proper superposition of nilpotent sensors
Sensor network design (5)

→ Signals of nilpotent sensors in damaged case measure slope!

**Results: Original frame**

![Graphs showing signal measurements for different sensors.]

**Results: Frame with additional hinges**

![Graphs showing signal measurements for different sensors with additional hinges.]

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**Technische Mechanik**

Mechatronik

Johannes Kepler Universität Linz
Sensor network design (6)

→ 9 nilpotent sensors are used; three for each floor

Structure with hinge: 2\textsuperscript{nd} floor - right - lower end

Thank you for your kind attention!